Motion in Brane World Models: The Bazanski Approach

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Abstract

Recently, path equations have been obtained for charged, spinning objects in brane world models, using a modified Bazanski Lagrangian. In this study, path deviation equations of extended objects are derived. The significance of moving extended objects in brane world models is examined. Motion in non- symmetric brane world models is also considered.

The Bazanski Approach in Riemannian Geometry

Geodesic and geodesic deviation equations can be obtained simultaneously by applying the action principle on the Bazanski Lagrangian [1]:

$$L = g_{\alpha\beta} U^{\alpha} \frac{D\Psi^{\beta}}{Ds}, \tag{1}$$

where $\frac{D}{Ds}$ is the covariant derivative. By taking the variation with respect to the deviation vector Ψ^{ρ} one obtains the geodesic equation. Taking the variation with respect to the unit tangent vector U^{ρ}, one obtains its geodesic deviation equation respectively :

$$\frac{dU^{\alpha}}{ds} + \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} U^{\mu} U^{\nu} = 0, \tag{2}$$

$$\frac{D^2\Psi^{\alpha}}{Ds^2} = R^{\alpha}_{.\beta\gamma\delta} U^{\beta} U^{\gamma} \Psi^{\delta}, \tag{3}$$

where $\{^{\alpha}_{\mu\nu}\}$ is the Christoffel symbol of the second kind $R^{\alpha}_{\beta\gamma\delta}$ is the Riemann-Christoffel curvature tensor.

It is worth mentioning that the Bazanski approach has been successfully applied in geometries different from the Riemannian one [2],[3]. Also, the Lagrangian (1) can be amended to describe path and path deviation equations of spinning charged particles [4] by introducing the following Lagrangian:

$$L = g_{\alpha\beta} U^{\alpha} \frac{D\Psi^{\beta}}{Ds} + (\frac{e}{m} F_{\alpha\beta} U^{\beta} + \frac{1}{2m} R_{\alpha\beta\gamma\delta} S^{\gamma\delta} U^{\alpha}) \Psi^{\mu} \tag{4}$$

to give

$$\frac{dU^{\alpha}}{ds} + \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} U^{\mu} U^{\nu} = \frac{e}{m} F^{\mu}_{.\nu} U^{\nu} + \frac{1}{2m} R^{\alpha}_{.\mu\nu\rho} S^{\nu\rho} U^{\mu}. \tag{5}$$

Its spinning charged deviation equation becomes:

$$\begin{aligned} \frac{D^2\Psi^{\alpha}}{Ds^2} &= R^{\alpha}_{.\mu\nu\rho} U^{\mu} U^{\nu} \Psi^{\rho} + \frac{e}{m} (F^{\alpha}_{.\nu} \frac{D\Psi^{\nu}}{Ds} + F^{\alpha}_{.\nu;\rho} U^{\nu} \Psi^{\rho}) \\ &+ \frac{1}{2m} (R^{\alpha}_{.\mu\nu\rho} S^{\nu\rho} \frac{D\Psi^{\nu}}{Ds} + R^{\alpha}_{\mu\nu\lambda} S^{\nu\lambda}_{.;\rho} U^{\mu} \Psi^{\rho} + R^{\alpha}_{\mu\nu\lambda;\rho} S^{\nu\lambda} U^{\mu} \Psi^{\rho}), \end{aligned} \tag{6}$$

where $F^{\mu}_{.\nu}$ is the electromagnetic field tensor and $S^{\gamma\delta}$ is the spin tensor .

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The Bazanski Approach in Brane World Models

It is well known that in the Brane world scenario our universe can be described in terms of a 4+N dimensional, with $N \geq 1$ and the 4D space-time part of it is embedded in a 4+N manifold [5]. Accordingly, the bulk geodesic motion is observed by a four dimensional observer to reproduce the physics of 4D space-time [6]. Consequently, it is vital to derive the path and the path deviation equations for a test particle on a brane using the following Lagrangian [7] :

$$L = g_{\mu\nu}(x^{\rho}, y)U^{\mu}\frac{D\Psi^{\nu}}{Ds} + f_{\mu}\Psi^{\mu}, \tag{7}$$

where $g_{\mu\nu}(x^{\rho}, y)$ is the induced metric and$f_{\mu} = \frac{1}{2}U^{\rho}U^{\sigma}\frac{\partial g_{\rho\sigma}}{\partial y}\frac{dy}{ds}U_{\mu}$ describes a parallel force due to the effect of non-compactified extra dimension. The variation of the Lagrangian gives [8]:

$$\frac{dU^{\mu}}{ds} + \begin{Bmatrix} \mu \\ \alpha\beta \end{Bmatrix} U^{\alpha}U^{\beta} = (\frac{1}{2}U^{\rho}U^{\sigma} - g^{\rho\sigma})\frac{\partial g_{\rho\sigma}}{\partial y}\frac{dy}{ds}U^{\mu}. \tag{8}$$

As in the brane world models, one can express $\frac{1}{2}\frac{\partial g_{\rho\sigma}}{\partial y}$ in terms of the extrinsic curvature $\Omega_{\rho\sigma}$ i.e. $\Omega_{\alpha\beta} = \frac{1}{2}\frac{\partial g_{\rho\sigma}}{\partial y}$ [9]. Thus, equation (8) becomes:

$$\frac{dU^{\mu}}{ds} + \begin{Bmatrix} \mu \\ \alpha\beta \end{Bmatrix} U^{\alpha}U^{\beta} = 2(\frac{1}{2}U^{\mu}U^{\sigma} - g^{\mu\sigma})\Omega_{\rho\sigma}\frac{dy}{ds}U^{\rho}. \tag{9}$$

And its corresponding deviation equation is

$$\frac{D^2\Psi^{\alpha}}{Ds^2} = R^{\alpha}_{.\mu\nu\rho}U^{\mu}U^{\nu}\Psi^{\rho} + (U^{\alpha}U^{\sigma}U^{\nu})\Omega_{\sigma\nu}\partial y\frac{dy}{ds})_{;\rho}\Psi^{\rho} + 2((\frac{1}{2}U^{\alpha}U^{\sigma} - g^{\alpha\sigma})\Omega_{\rho\sigma}\frac{dy}{ds}U^{\rho})_{;\delta}\Psi^{\delta} \tag{10}$$

Also, applying the bazanski approach in brane world models, we obtain the path and path deviation equations for a spinning charged object, respectively

$$\frac{dU^{\alpha}}{ds} + \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} U^{\mu}U^{\nu} = \frac{e}{m}F^{\alpha}_{\nu}U^{\nu} + \frac{1}{2m}R^{\alpha}_{\beta\mu\nu}S^{\mu\nu}U^{\beta} + 2(\frac{1}{2}U^{\alpha}U^{\rho} - g^{\alpha}\rho)\Omega_{\rho\delta}\frac{dy}{ds}U^{\delta} \tag{11}$$

and

$$\frac{D^2\Psi^{\alpha}}{Ds^2} = R^{\alpha}_{.\mu\nu\rho}U^{\mu}U^{\nu}\Psi^{\rho} + \frac{1}{2m}(R^{\alpha}_{.\mu\nu\rho}S^{\nu\rho}\frac{D\Psi^{\mu}}{Ds} + R^{\alpha}_{\mu\nu\lambda}S^{\nu\lambda}_{.;\rho}U^{\mu}\Psi^{\rho} + R^{\alpha}_{\mu\nu\lambda;\rho}S^{\nu\lambda}U^{\mu}\Psi^{\rho})$$
$$+ \frac{e}{m}(F^{\alpha}_{\rho}\Psi^{\rho} + F^{\alpha}_{\rho}\frac{D\Psi^{\rho}}{Ds}) + (\frac{1}{2}U^{\alpha}U^{\sigma}U^{\nu})\Omega_{\sigma\nu}\frac{dy}{ds})_{;\rho}\Psi^{\rho} + (\frac{1}{2}U^{\alpha}U^{\sigma} - g^{\alpha\sigma})\Omega_{\sigma\nu}\frac{dy}{ds}\frac{D\Psi^{\nu}}{Ds}. \tag{12}$$

Thus, equations (11) and (12) are derived from the following Lagrangian

$$L = g_{\mu\nu}(x^{\rho}, y)U^{\mu}\frac{D\Psi^{\nu}}{Ds} + 2(\frac{1}{2m}R_{\mu\nu\rho\sigma}S^{\rho\sigma}U^{\nu} + \Omega_{\rho\sigma}\partial yU^{\rho}U^{\sigma}U_{\mu}\frac{dy}{ds})\Psi^{\mu}, \tag{13}$$

Path & Path Deviation Equations of Non-Symmetric Geometries in Brane World Models

Path equation and Path deviation equations in Brane World Models defined in non-symmetric geometries can be obtained by suggesting the following Lagrangian:

$$L = \mathbf{g}_{\mu\nu} U^{\mu} \frac{D\Psi^{\nu}}{D\tau} + \lambda f_{[\mu\nu]} U^{\mu} \Psi^{\nu} + \frac{1}{2} U^{\alpha} U^{\beta} U^{\rho} \frac{\partial \mathbf{g}_{\alpha\beta}}{\partial s} \frac{dy}{ds}, \tag{14}$$

where $\mathbf{g}_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]}$,$\lambda$ is a parameter and, $f_{[\mu\nu]} = \hat{A}_{\mu,\nu} - \hat{A}_{\nu,\mu}$ is a skew symmetric tensor related to the Yukawa force [10].

Applying the Bazanski approach we obtain the path equation

$$\frac{dU^{\alpha}}{ds} + \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} U^{\mu} U^{\nu} = \lambda \mathbf{g}^{\alpha\mu} f_{[\mu\nu]} U^{\nu} + \mathbf{g}^{\alpha\sigma} g_{[\nu\sigma];\rho} U^{\nu} U^{\rho} + (\frac{1}{2} U^{\rho} U^{\sigma} - \mathbf{g}^{\rho\sigma}) \frac{\partial \mathbf{g}_{\rho\sigma}}{\partial y} \frac{dy}{ds} U^{\mu}. \tag{15}$$

and its path deviation equation:

$$\frac{D^2\Psi^{\alpha}}{Ds^2} = R^{\alpha}_{.\mu\nu\rho} U^{\mu} U^{\nu} \Psi^{\rho} + 2\mathbf{g}^{\sigma\alpha} (g_{[\nu[\sigma];\rho]}) \frac{D\Psi^{\nu}}{Ds} U^{\rho} + \lambda (f^{\alpha}_{.\nu} \frac{D\Psi^{\nu}}{Ds} + f^{\alpha}_{.\nu;\rho} U^{\nu} \Psi^{\rho}).$$
$$+ ((\frac{1}{2} U^{\alpha} U^{\rho} - \mathbf{g}^{\alpha\rho}) \frac{\partial \mathbf{g}_{\delta\rho}}{dy} U^{\delta} \frac{dy}{ds})_{;\nu} \Psi^{\nu} + (\frac{1}{2} U^{\alpha} U^{\mu} \frac{\partial \mathbf{g}_{\mu\nu}}{dy} \frac{dy}{ds}) \frac{D\Psi^{\nu}}{Ds} \tag{16}$$

The Bazanski Approach in Curved Clifford Space

It is well known that extended objects can be expressed by p-branes[11]. This type of representation is defined in curved Clifford Space. The advantage of this space is to show that extended objects are purely poly-geodesics satisfying a metamorphic transformation.[12].

Thus, we suggest the following Lagrangian:

$$L = g_{AB} P^{\alpha} \frac{\nabla \phi^{\beta}}{\nabla \tau} + \frac{1}{2} S_{\alpha\beta} \frac{\nabla \phi^{\alpha\beta}}{\nabla \tau}, \tag{17}$$

such that

$$\frac{\nabla \phi^{\alpha}}{\nabla \tau} = \frac{d\phi^{\alpha}}{d\tau} + \Gamma^{\alpha}_{\beta\rho} U^{\rho} \phi^{\beta} + \frac{1}{2m} \hat{R}^{\alpha}_{.\beta\gamma\delta} S^{\gamma\delta} \phi^{\beta} + \frac{1}{2m} S^{\omega}_{\gamma} ({\Xi_{\mu\omega}}^{\alpha\sigma} U^{\mu} + \frac{1}{2m} S^{\rho\beta} {\Omega_{\rho\beta\omega}}^{\alpha\sigma}),$$

where $\Gamma^{\alpha}_{\beta\rho}$ is the Cartan connection, $\hat{R}^{\alpha}_{.\beta\gamma\delta}$ is the Cartan curvature and the two other metamorphic connections ${\Xi_{\beta\rho}}^{\alpha\delta}$&$\Omega\beta\rho\gamma\ {}^{\alpha\delta}$ existed due to the presence of bivector quantities. By taking variation with respect to its deviation vector ϕ^{δ} we get

$$\frac{\nabla U^{\alpha}}{\nabla \tau} = 0 \tag{18}$$

and taking the variation with respect to its deviation bivector $\phi^{\delta\rho}$ we obtain

$$\frac{\nabla S^{\alpha\beta}}{\nabla \tau} = 0 \tag{19}$$

In the case of a charged particle described by a polyvector we can find that the corresponding linear momentum equation becomes [12]

$$\frac{\nabla U^{\alpha}}{\nabla \tau} = \frac{e}{m} F^{\alpha}_{\beta} U^{\beta} \tag{20}$$

and its angular momentum equation takes the following form

$$\frac{\nabla S^{\alpha\beta}}{\nabla \tau} = F^{\alpha}_{\nu} S^{\nu\beta} - F^{\beta}_{\nu} S^{\nu\alpha} \tag{21}$$

We suggest the following Lagrangian to derive (20) and (21)

$$L = g_{\mu\nu} P^{\mu} \frac{\nabla \phi^{\nu}}{\nabla \tau} + \frac{1}{2} S_{\mu\nu} \frac{\tau \phi^{\mu\nu}}{\nabla \tau} + F_{\mu\nu} U^{\nu} \phi^{\mu} + \frac{1}{2} (F_{\mu\rho} S^{\rho}_{\nu} - F_{\nu}{}^{\rho} S_{\rho\mu}) \phi^{\mu\nu}. \tag{22}$$

If we put the flavor of Kaluza-Klein in curved Clifford space for combining gravity and electromagnetism, i.e, increasing the spatial dimension by a compacted one, then equations (20),(21) can be derived from the following Lagrangian:

$$L = g_{AB} P^{A} \frac{\nabla \phi^{B}}{\nabla \hat{\tau}} + \frac{1}{2} S_{AB} \frac{\nabla \phi^{AB}}{\nabla \hat{\tau}}, \tag{23}$$

where $A = 1, 2, 3, 4, 5$ to become

$$\frac{\nabla U^{\alpha}}{\nabla \hat{\tau}} = 0 \tag{24}$$

and

$$\frac{\nabla S^{\alpha\beta}}{\nabla \hat{\tau}} = 0 \tag{25}$$

which means that the path of a charged spinning particle defined in Riemannain geometry behave like as a test particle in 5D curved Clifford space.

Discussion and Concluding Remarks

In this study, we have derived path and path deviation equations for test particles and spinning charged test objects in Brane World Models from one single Lagrangian. Also, the procedure has been used to derive path and path deviation equations for a test particle existing in non-symmetric theory of gravity and examined how the extrinsic curvature term would be amended due to the existence of non-symmetric terms of gravitational field. Thus, we can find that the added term appeared in the extrinsic curvature of non-symmetric part may be connected to the spin. Finally, we have dealt with extended objects as p-branes defined in curved Clifford space [13]. This type of space has a deeper understanding of physics, i.e., quantities in nature could be defined by polyvectors. From this perspective we have developed its corresponding Bazanski Lagrangian to include deviation bi-vectors together with deviation vectors. The importance of this approach is that we can derive from one Lagrangian(17) two simultaneous quantities responsible for conservation of momentum (18) and angular momentum (19) respectively. Using this mechanism, we have found that the Dixon-Souriau equations in Riemannian geometry could be seen like equations for charged object and their angular momentum part

defined in curved clifford spaces. In addition, if we have the flavor of Kaluza-Klein of unifying gravity and electromagnetism in curved Clifford space, we must increase the dimension of this space by an extra compacted dimension to preserve the conservation of charges. Consequently, from equations (24) and (25) we can find that charged spinning particles behave like test particles defined in higher dimensional curved clifford space. To conclude this study, we must take into consideration that unification processes can be achieved if we increase the number of dimensions and extend the geometries to geometrize all quantities that appeared in our approach.

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